In a nutshell: Newton's method for finding extrema in n dimensions

Given a continuous and differentiable real-valued function f of a vector variable with one initial approximation of a minimum $\mathbf{x}_0$ where the Hessian, that is, the Jacobian of the gradient, of f at that point, $\mathbf{H}(f)(\mathbf{x}_0) = \mathbf{J}(\nabla f)(\mathbf{x}_0)$ is invertible. If the gradient is already the zero vector, we are done. This algorithm uses iteration, Taylor series and solving systems of linear equations to approximate a minimum.

Parameters:

$\varepsilon_{\text{step}}$	The maximum error in the value of the minimum cannot exceed this value.
ε_{abs}	The difference in the value of the function after successive steps cannot exceed this value.
N	The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
3. Solve $\mathbf{J}(\nabla f)(\mathbf{x}_k)\Delta\mathbf{x}_k = -\nabla f(\mathbf{x}_k)$ for $\Delta\mathbf{x}_k$ where $\mathbf{J}(\nabla f)(\mathbf{x})$ is the Jacobian of ∇f evaluated at the point $\mathbf{x}$.
   Let $\mathbf{x}_{k+1} \leftarrow \mathbf{x}_k + \Delta\mathbf{x}_k$.
   a. If $\mathbf{x}_{k+1}$ has any entries that are not finite floating-point numbers, return signalling a failure to converge.
   b. If $\|\mathbf{x}_{k+1} - \mathbf{x}_k\|_2 < \varepsilon_{\text{step}}$ and $|f(\mathbf{x}_{k+1}) - f(\mathbf{x}_k)| < \varepsilon_{\text{abs}}$, return $\mathbf{x}_{k+1}$. A sufficient condition that this is a minima is that the Hessian is positive definite (all positive eigenvalues).
4. Increment k and return to Step 2.

Convergence

If h is the error, it can be show that the error decreases according to $O(h^2)$. This technique is not guaranteed to converge if there is a minimum, for the Hessian could be near singular, causing the next approximation to be arbitrarily far from the previous approximation.